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References and Applicable Documents

1. https://exdci.eu
2. http://www.exascale.org/bdec/
3. http://ec.europa.eu/information_society/newsroom/cf/dae/document.cfm?doc_id=8937TBC
4. https://ec.europa.eu/digital-single-market/en/news/workshop-mathematics-and-digital-science

List of Acronyms and Abbreviations

AI	Artificial Intelligence
BDEC	Big Data and Extreme Scale Computing
CIRM	Centre Internationale des Rencontres Mathématiques
DFG	Deutsche Forschungsgemeinschaft
EC	European Commission
ERC	European Research Council
EIC	European Innovation Council
EU	European Union
EURO	European Operational Research Societies
EXDCI	European eXtreme Data and Computing Initiative
HPC	High Performance Computing
MFO	Mathematisches Forschungsinstitut Oberwolfach
MSCA	Marie Skłodowska Curie Actions
ML	Machine Learning
PPP	Public Private Partnership
TRL	Technology Readiness Level
US/USA	United States of America

Executive Summary

In today's information-rich and data-rich society, the need for new mathematics and its coupling with High Performance Computing and Machine Learning is omnipresent.

This report presents a survey and a number of one-to-one interviews that were held with eminent representatives of the European mathematics and machine learning communities. The objective of the report is to outline important pathways and topics for future mathematics in Europe and how mathematical research could be coupled with machine learning and HPC.

The conclusions are: (1) that the ERC should remain the principal source of funding for mathematics, but with broader reach. In particular, smaller grants should be made available to younger researchers. (2) It is by emphasizing and actively supporting community-building that we will be able to enrich and preserve European mathematics, and thus place Europe at the forefront of the imminent AI revolution. This should take the form of co-funding through PPPs that build upon existing teams and institutions. (3) Define and fund one big mathematical flagship project based on the topics exposed in this document.

1. Introduction

Numerous national and EC reports in the past have underlined the importance of mathematics for the future of societies and technologies (see Appendix for the list). More recently, we are confronted by the "data deluge" and the impressive uprising of artificial intelligence, in the form of Machine Learning (ML), as a means for analysing these data and extracting knowledge from them. To address these, mathematics needs to be supported, in a broad sense, since a broad range of branches of mathematics (both applied and pure), statistics and optimisation are all required for mastering this new world.

The BDEC community, after addressing the issue of convergence of data and HPC in its extensive report (bit.ly/bdec-pathways), is now turning towards "scientific machine learning" (SML) that aims to combine HPC and machine learning with modelling of physical systems. This coupling requires going beyond the "simple" convergence of data and HPC, since HPC-like workflows are now indicated for performing SML at different points in the data acquisition process, all along the data pathway, from edge devices to centralised computing centres.

2. Work Method

Our approach was based on questions that we formulated and then posed to representatives of learned societies, mathematicians and computer scientists in Europe. We began by setting up a general survey on a specialised website. This survey was sent out to a long list of over 100 contacts (both personal and institutional—see Appendix) among which we mention, in particular,

- Learned societies.
- Academic and research institutions.
- Industry.

The responses were collected automatically in a spreadsheet.

After the survey was terminated, we decided to follow up with several one-on-one interviews with selected individuals, some of whom may had already participated in the first phase. The aim of these interviews was to drill down deeper into some specifics of content and funding.

3. Survey and Results

3.1. Questions

1. **In which domain of mathematics are you active?**
2. **How do you fund your research?**
3. **How many non-EU research funds (approximately) has your group (the group that you are leading or you belong to) collected in the last 3 years (in FTE)?**
4. **How many EU research funds (approximately) has your group (the group that you are leading or you belong to) collected in the last 3 years (in FTE)?**
5. **Have you already been a member of consortium that prepared a rejected proposal for an EU funded research project?**
6. **Please indicate the actions where you have won at least one project as a leading or full partner.**
7. **Do you have any scientific cooperation with research partners from non-mathematical domains?**
8. **Do you find EU programs suitable for funding research in mathematics?**
9. **Are you aware of the document "Mathematics for Digital Science"?**
10. **Do you see any positive impact of the document "Mathematics for Digital Science"**
11. **Are you involved in any national or international body that is involved in preparations for future funding programs for research?**
12. **Have you ever been a referee for research projects?**
13. **If the EU were to propose mathematical content in the future calls (in addition to ERC calls), what are the specific topics that you would like to see?**

Our questions attempted to cover two major points: funding and content. That is:

- How is current research in Mathematics funded?
- If and how future mathematics should be funded at the EC level?
- What are the open problems, or important topics for such funding calls?

Some questions had a series of options as well as sub-questions. Please see the attached pdf document for full details and responses.

3.2. Responses

In spite of a very broad diffusion effort, as well as numerous "reminders" to please fill in the survey, the overall number of responses (12) was quite disappointing. This could be attributed to a number of factors:

1. The survey was too detailed.
2. The questions were quite difficult for certain members of the community, who are not familiar with funding and calls…
3. The community does not care that much about funding, and just wants to be "left in peace" to do their research.

Although we cannot make any claims of statistical significance, we have made the following observations from the responses:

1. Most of those who responded, 2/3, have used EU funds for their research.
2. The use of the MSCA funding stream is very low (1/10).
3. There is strong cooperation with other disciplines.
4. There is some adequation of EU-funding with mathematics research, thanks to the ERC.
5. There is no clear preference, or tendency, for future research directions.

For all the above reasons, we decided to perform follow-up interviews, one-on-one, with a selected list of eminent and active members of the community, some of whom had already filled in the survey. The results and analysis are presented in the next section.

4. One-on-one Interviews

Given the relative paucity of responses to the initial survey, it was decided to rather hold one-on-one interviews with chosen interviewees (see Appendix for the complete list). We also reduced the number of questions and modified their content to better reflect strategic aspects of future research and its funding.

Each interview consisted of filling in the questionnaire together and of having a general discussion with the interviewee.

4.1 Questions

For consistency, some of the questions of the survey were used in the interviews, but they were re-oriented to provide a more global strategy.

Q1. Is there a need for additional funding of (applied) math research at EU level?

Q2. What form should this funding take? Extra ERC money, RIA projects, others (flagship)?

Q3. What are the hot topics for research in the next 5 years? Can these be financed at national level, or should there be EU/EC funding?

Q4. How about interdisciplinary subjects (math + x), where x could be cybersecurity, health, natural disasters, biodiversity, etc. Should there be separate funding, or should mathematicians join such projects in existing societal challenge calls?

Q5. Math + Industry. Is there a need for additional, targeted EU funding?

Q6. Who is doing math for AI? Is it being funded sufficiently?

Q7. Is there a place/need for EU mathematical flagship projects like Human Brain, Quantum computing, High Performance computing etc.? If YES, what could be the main topic of such project?

4.2 Responses

The responses were very diversified and reflect faithfully the inhomogeneous nature of the community. However, the points raised are all pertinent and merit attention. In what follows, we have extracted and summarized the most relevant and consensual responses, rather than listing comprehensively the results of the interviews.

Math Funding

It is clear that the ERC remains the principal funding source for mathematics in Europe today. There is a general consensus that ERC grants are too big and there would be greater benefit from awarding a larger number of smaller grants, with higher chances to be selected. The experience of the majority of interviewees brought out the fundamental issue that too many top-quality proposals get rejected because there is not enough money. This is an immense waste of time both for researchers, who take huge efforts to prepare excellent proposals but do not receive funding, and for the referees who review lots of proposals which get rejected.

Another gap exists for young researchers who have enormous difficulty to enter the ERC. A possible solution would be some kind of "starting synergy" grant. Mathematics should figure explicitly in other sections of the ERC, and not only in PE1[1]. It is a pity that the EIC (European Innovation Council) does not accept projects with lower TRL's that would enable an active participation of mathematicians.

Existing RIA funding is not considered to be well-adapted to mathematics, since it usually positions mathematics as a service, and not as an autonomous, or semi-autonomous research topic. This restriction does not allow good mathematics to flourish.

Interdisciplinary topics are at the core of applied mathematics research. An effort to include mathematics in research should be imperative, like the humanities is often required. Changing research practices across fields to include mathematicians by default is important.

A special program within EuroHPC, or, more generally, setting up a European technology Platform for Mathematical Modelling, Simulation and Optimisation in a Data-rich environment with the explicit support of the EU within the new framework program, Horizon Europe.

Math for AI

As far as the topic of AI/ML is concerned, the sentiment is that it is done mostly by computer scientists and there are too few mathematicians involved. However, history and experience show that major algorithmic advances originate invariably from mathematical research. There is thus a clear need to attract more mathematicians to this topic, because if we do not succeed then all the research will be done (and *is* being done to a large extent -at the moment) by Google, Facebook and Amazon.

There is an urgent need for community building in Europe around this theme, to slow down and reduce the actual brain-drain. This loss of our best young minds is largely due to the better academic and financial opportunities available, in particular in the US, but also (more and more) in Asia.

[1] There are 10 ERC evaluation panels, PE1 to PE10, in Physical Sciences and Engineering. All areas of mathematics, both pure and applied, are evaluated by the first panel, PE1.

Now a lot of math people claim to do math for AI and indeed some applied and even basic math research can be useful for ML without being labelled as such. However, this is not the same as deliberately deciding to focus on ML issues with the mathematical methodology. Such focused research directions should be encouraged and supported, but not at the expenses of other research directions.

Math for HPC

HPC is the foundation, without which future technology and societal challenges will not be able to be addressed using models and AI. It is for this reason that mathematics for HPC is a fundamental investment in the value chain that goes from raw data, through models and ML, to actionable knowledge. This cycle has been recognized as the basis for scientific discovery—see also the BDEC report (bit.ly/bdec-pathways), to which the EXDCI project has contributed. Also, excellent European representation in the BDEC international workshops can be observed by consulting the site www.exascale.org where BDEC is presented and all workshops are presented in detail– all the material is freely available. All the interviewees were in agreement with this view of mathematics for HPC.

Hot Topics

Some interviewees are firmly against a choice of topics that are "imposed" on the mathematics community. The initiative should be left to the researchers themselves, in a bottom-up fashion, as is done for example by the DFG in Germany.

Others have proposed the following topics (see also "Maths + X" below):

- Mathematics and Algorithms for ML:
  - Robustness of ML algorithms
  - Ill-posedness and making ML well-posed
  - Functional Spaces for classification problem
  - High dimensional analysis: function spaces, probability distributions, flows, PDEs related to common ML and DL architectures
  - Specific issues: large time asymptotics for gradient flows, density estimation in high dimension
  - Relationships with statistical physics
  - Physics-informed AI.
- Next-generation model inversion.
- Automated hypothesis generation and causal discovery.
- Enabling AI computing regimes and supporting hardware.
- Learning algorithms for novel computing regimes (quantum, exascale).
- Robust learning for science (anomaly detection, uncertainty quantification).
- Power-constrained learning and power-efficient learning.
- Algorithms for HPC: new linear algebra solvers, new preconditioners, mixing 16 and 64 bit calculations, use of GPUs rather than CPUs.
- Singular systems.
- Multiscale problems.
- Topological Data and Image Analysis.

- Cryptography.
- Non-convexity in optimisation using new tools, such as real algebraic geometry.
- Commutative and non-commutative algebra in Quantum Computing (quantum information), and analysing links with combinatorial optimization.
- High Performance computing for combinatorial optimization.

Math + X

This has been very successful in the US, where the Simons Foundation (https://www.simonsfoundation.org/) uses a matched funding instrument for funding chairs: "Through a matching grant for endowment, the Math+X program creates joint Chairs, each shared equally between a mathematics department and a partner department. The grants include substantial operating funds to support activities shared between the two departments.[2]" The EC could emulate this, and even do better, based on its experience of PPPs that would engage existing research and academic institutions (including visitor centres such as MFO, CIRM, etc.) in a joint funding with the EC. The enormous advantage of this approach is that it is bottom-up, and builds upon existing programs, competencies, human resources, without making competitive selection that, in the end, will always restrict the number of recipients and beneficiaries.

Math + Industry

Math for digital twins is a big issue and a big opportunity, and industry is heavily in need of this.

It has to come from both sides: mathematicians willing to cooperate with industry, realizing that this will also lead to new mathematical challenges and new mathematics. And industry realizing they cannot do without mathematics, since the world has become so complex that only mathematics can help in mastering this.

Here again, joint funding would be an excellent approach.

Flagship projects

Big projects involving mainly diverse mathematical communities and with non-mathematical groups, like computer science, engineering, life science attached, will certainly be a historical milestone for European mathematics. Appropriate envelopes of the topics mentioned above could be the main content of such projects.

[2] https://www.simonsfoundation.org/mathematics-physical-sciences/mathx-encouraging-interactions/

5. Conclusions and Recommendations

There is general consensus concerning the need for mathematics in Europe that is manifested by the numerous reports (see Appendix for the list) that have been published in the last 8 years. Project funding is a vital stimulus, but should remain within the framework of the ERC. To achieve this, the ERC's remit should be broadened to allow smaller grant amounts and inclusion of mathematics in other Evaluation Panels.

There are a number of priority research areas that could be proposed. These cover *mathematics for machine learning*, *mathematics for industry* and *mathematics for high performance computing*. These could be the object of dedicated calls, possibly within the existing frameworks of RIAs and CSAs, though community-building efforts would seem to be what is most often requested. This could be achieved via Public-Private-Partnerships between the EC and existing academic and research institutions, based on co-funding.

Defining and funding one big mathematical flagship project enveloping the topics exposed in this document, with many mathematical groups involved would have a historical positive impact on European mathematics.

Funding and structures should *create the conditions* in which mathematics can thrive and whose collaborations will enable the development of ambitious European projects.

In conclusion, to preserve Europe's mathematical heritage and prepare society's future, we should propose attractive research incentives and support to all mathematicians, especially younger ones.

Recommendations:

R1 – broaden the ERCs remit, to enable larger number of small grants to younger mathematicians and the appearance of mathematics in other panels.

R2 – launch dedicated research funding programs in Mathematics for AI, Mathematics for Industry and Mathematics for HPC.

R3 – set up a generous co-funding, or matched-funding program based on Public-Private-Partnerships, where the EC creates the conditions for bottom-up research projects, building upon teams that exist already in industry, universities and research institutes.

Appendix – Survey Questions and Result Analysis

The full result analysis can be found in the attached pdf document.

Appendix – List of Institutional Survey Addressees

Name	*Country*	*Type*	
SMF	FR	LS	Societe Mathematique de France
SMAI	FR	LS	Societe Mathematiques Appliquees et Industrielles
SFdS	FR	LS	Societe Francaise de Statistiques
SIF	FR	LS	Societe Informatique de France
FMJH	FR	LS	Fondation Mathematique Jacques Hadamard
FSMP	FR	LS	Fondation Sciences Mathematiques de Paris
ANR	FR	NFA	Agence Nationale de la Recherche
DFG	DE	NFA	Deutsche Forschungsgemeinschaft
Max Planck	DE	NRI	
INDAM	IT	LS	Instituto Nazional di Alta Mathematica
Imperial College	UK	U	
EPSRC	UK	NFA	Engineering Physical Sciences Research Council
FWF	AT	NFA	Fonds zur Forderung der wissenschaftlichen Forschung
NWO	NL	NFA	Nederlands Organisatie voor Wettenschappelijk Onderzoek
ICMAT	ES	LS	Instituo de Ciencias Matematicas
HAS	HU	LS	Hungarian Academy of Science
EMS	EU	LS	European Mathematical Society
INSMI-CNRS	FR	NRI	Institut National des Sciences Mathematiques
EU-MATHS-IN	EU	EUP	European Network Mathematics Indistry and Innovation
ERCIM	EU	LS	European research Consortium for Informatics and Mathematics
IMI-BAS	BG	LS	Institute of Mathematics and Informatics Bulgarian Academy of Sciences
Royal Stat Society	UK	LS	
European Society for Mathematical and Theoretical Biology	EU	LS	
ENBIS-European Network for Business and Industrial Statistics	EU	LS	
CWI	NL	NRI	Centrum Wiskunde & Informatica
INRIA	FR	NRI	Institut National de Recherche en Sciences et Technologies du Numerique

LS	Learned Society
NFA	National Funding Agency
U	University
NRI	National Research Institute
EUP	EU Project

Appendix – List of Interviewees

First name	Last name	Affiliation
Olivier	PIRONNEAU	French Academy of Sciences
Stephane	MALLAT	College de France, Paris.
Pascal	AUSCHER	National Institute of Mathematics, CNRS. Paris.
Jean-Stephane	DHERSIN	National Institute of Mathematics, CNRS. Paris.
Wil	SCHILDERS	EU-MATHS-IN
Jean Bernard	LASSERRE	LAAS-CNRS, Toulouse.
Immanuel	BOMZE	EURO president (https://www.euro-online.org/)
Jean-Pierre	BOURGUIGNON	ERC (past president 2014-2019)
Klavdija	KUTNAR	Rector of University of Primorska, Slovenia
Mirjam	DUER	University of Augsburg

Appendix – Reports on Impact of Mathematics

The new intricate relation mathematics entertains with production and services has led to numerous reports evaluating its impact on the economy. This was the purpose of the report "*Measuring the Economic Benefits of Mathematical Science Research in the UK*"[3] in 2012 and of the report *"The Mathematical sciences and their value for the Dutch economy"*[4] in 2014. They were followed by similar studies in France[5] in 2015 and in Spain[6] in 2019. A more open view, based on the same basis, was the source of the report "*THE ERA OF MATHEMATICS, An Independent Review of Knowledge Exchange in the Mathematical Sciences*"[7] in 2018. It inspired the Japanese report *"The Coming Era of Mathematical Capitalism – How the Power of Mathematics Changes our Future"*[8] in 2019. A number of reports were also commissioned in the USA, both by the NSF and the DOE, as well as by the National Academy of Sciences. Any of these documents would have been unthinkable 20 years ago.

[3] Report *"Measuring the Economic Benefits of Mathematical Science Research in the UK"* also commissioned to Deloitte by the Engineering and Physical Sciences Research Council (EPSRC) accessible at https://epsrc.ukri.org/newsevents/pubs/deloitte-measuring-the-economic-benefits-of-mathematical-science-research-in-the-uk/

[4] Report *"The Mathematical sciences and their value for the Dutch economy"* commissioned to Deloitte by the Platform Wiskunde Nederland accessible at euro-math-soc.eu/system/files/uploads/DeloitteNL.pdf .

[5] Report *"Étude de l'impact socio-économique des Mathématiques en France"* accessible on the AMIES website.

[6] Report *"Socio-economic impact of mathematical research and mathematical technology in Spain"* accessible on the institucionales.us.es website.

[7] Report *"THE ERA OF MATHEMATICS An Independent Review of Knowledge Exchange in the Mathematical Sciences"* under the leadership of Philip BOND accessible at https://epsrc.ukri.org/newsevents/pubs/era-of-maths/ . The report was also commissioned by the EPSRC

[8] The METI and MEXT Report *"The Coming Era of Mathematical Capitalism – How the Power of Mathematics Changes our Future"* has an English summary accessible at https://www.meti.go.jp/english/press/2019/0326_004.html ; the full Japanese version can be accessed at https://www.meti.go.jp/press/2018/03/20190326005/20190326005-2.pdf .